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Domain walls of $D = 8$ gauged supergravities and their $D = 11$ origin

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Abstract: Performing a Scherk-Schwarz dimensional reduction of $D = 11$ supergravity on a three-dimensional group manifold we construct five $D = 8$ gauged maximal supergravities whose gauge groups are the three-dimensional (non-)compact subgroups of $\mathrm{SL}(3,\mathbb{R})$. These cases include the Salam-Sezgin SO(3) gauged supergravity. We construct the most general half-supersymmetric domain wall solutions to these five gauged supergravities. The generic form is a triple domain wall solution whose truncations lead to double and single domain wall solutions. We find that one of the single domain wall solutions has zero potential but nonzero superpotential.
Upon uplifting to 11 dimensions each domain wall becomes a purely gravitational 1/2 BPS solution. The corresponding metric has a $7 + 4$ split with a Minkowski 7-metric and a 4-metric that corresponds to a gravitational instanton. These instantons generalize the SO(3) metric of Belinsky, Gibbons, Page and Pope (which includes the Eguchi-Hanson metric) to the other Bianchi types of class A.

Keywords: Solitons Monopoles and Instantons, Superstring Vacua, Gauge Symmetry.

Contents

1. Introduction

Gauged supergravities have become the focus of recent research due to a variety of reasons. Most applications are related to the fact that gauged supergravities contain a nonzero potential for the scalar fields. This potential, which behaves as an effective cosmological constant, allows for interesting vacuum solutions such as de Sitter and anti-de Sitter spacetimes or (half-supersymmetric) domain wall solutions. The anti-de Sitter vacua are important in the context of the AdS/CFT correspondence [1] while the possibility of a de Sitter vacuum is of interest due to recent astronomical observations [2]. This has triggered a search for de Sitter vacuum solutions in string theory (see, e.g., [3, 4]). Domain wall solutions have applications to the DW/QFT correspondence [5, 6], the braneworld scenario [7, 8] and cosmological models (see, e.g., [9, 10]).

We are particularly interested in domain wall solutions in $D = 5$ dimensions in connection with the search for a supersymmetric braneworld scenario. Finding the most general 1/2 BPS domain wall solution in $D = 5$ dimensions is a difficult task due the fact that the scalar potential is a complicated function of the many scalars that are present in the theory. To learn more about the $D = 5$ situation, it is instructive to study the simpler case of general 1/2 BPS domain wall solutions of maximal supergravities in higher dimensions $D \leq 11$. The first nontrivial example is $D = 9$ and this case was already discussed in [11]–[14]. The aim of this paper is to study the situation in $D = 8$ dimensions.

The standard $D = 8$ gauged maximal supergravity is the SO(3)-gauged theory of Salam and Sezgin [15]. They constructed this theory by applying what we will call a Scherk-Schwarz 2 (SS2) reduction procedure [16] to $D = 11$ supergravity. The SS2 procedure corresponds to a reduction on a group manifold where one uses a *symmetry of the compactification manifold* to give a specific dependence of the $D = 11$ fields on the compactification coordinates. This dependence is such that, although the $D = 11$ fields depend on them, the resulting $D = 8$ action does not. In contrast, there is also a so-called SS1 procedure [17] where the higher-dimensional fields acquire a dependence on the compactification coordinates by using a *global, internal symmetry* of the higher-dimensional theory, such as the $SL(2, \mathbb{R})$ symmetry of Type IIB supergravity.

In this paper we repeat and generalize the analysis of [15] to a group manifold corresponding to an arbitrary Lie algebra, instead of SO(3) only. We will show that the standard Bianchi classification of three-dimensional Lie algebras, see e.g. [18], leads to five cases, including the S^3 group manifold used in [15]. Performing a SS2 reduction of $D = 11$ supergravity with respect to these five distinct group manifolds we construct five gauged maximal supergravities where the gauge groups are the three-dimensional (non-)compact subgroups of $SL(3, \mathbb{R})$. These cases include the Salam-Sezgin SO(3) gauged supergravity. The other four cases, which involve the gauge groups $SO(2, 1)$, ISO(2), $ISO(1, 1)$ and the Heisenberg group, can be obtained by analytic continuation and/or generalized Inönü-Wigner contractions of the Salam-Sezgin theory.

We point out that the Bianchi classification allows for five more cases where the gauge groups are three-dimensional non-compact subgroups of $GL(3, \mathbb{R}) = SL(3, \mathbb{R}) \otimes SO(1, 1)$. Due to the extra $SO(1, 1)$-factor, the group manifold isometries are only symmetries of the equations of motion but not of an action. Therefore the SS2 procedure leads in these cases to gauged supergravities whose equations of motion cannot be integrated to an action. Two of these theories contain a free mass parameter. In the limit that this mass parameter goes to zero one recovers two of the gauged supergravities that do have an action. To distinguish between supergravities having an action or having no action we will denote the ones with an action as class A supergravities and the ones without an action as class B supergravities, in accordance with the Bianchi classification.

We will present a 1/2 BPS triple domain wall solution that solves the equations of motion corresponding to each of the different class A $D = 8$ gauged supergravities. The truncation of this triple domain wall solution to a single domain wall solution reproduces results that partly are already available in the literature. In particular, we find a new domain wall solution with zero potential but nonzero superpotential. We discuss the uplifting of the triple domain wall solution to $D = 11$ dimensions and show that, after uplifting, it becomes a purely gravitational 1/2 BPS solution. In each case the $D = 11$ metric has a 7+4 split with a Minkowski 7-metric and a 4-metric that can be identified with a $D = 4$ gravitational instanton [19]. These instantons generalize the metric of Belinsky, Gibbons, Page and Pope (which includes the Eguchi-Hanson metric) to the other (class A) Bianchi types.

The organization of this paper is as follows: in section 2 we perform the SS2 reduction of $D = 11$ supergravity leading to the two classes of $D = 8$ gauged supergravities mentioned above. In section 3 we give the action and transformation rules of the five different $D = 8$

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class A gauged supergravities. The triple domain wall solution is presented in section 4. Subsequently, in section 5 the triple domain wall solution is uplifted to $D = 11$ dimensions and linked to known solutions in the literature. Finally, in the Conclusions we discuss several extensions and open issues. This paper contains four appendices. Our conventions are given in appendix A. In appendix B we give a few details of $D = 11$ supergravity. Appendix C contains the details of the (bosonic) action and supersymmetry rules of the class A $D = 8$ gauged supergravities. Finally, in appendix D we collect some basic material on the classification of three-dimensional Lie algebras.

2. Reduction on a group manifold

In this section we perform the reduction of $D = 11$ supergravity over a three-dimensional group manifold to $D = 8$ dimensions. The group manifold reduction procedure generally gives rise to gauged supergravities, where the structure constants of the gauge group $\mathcal{G}$ are provided by the group manifold.

In the case at hand the group manifolds are three-dimensional. The prime example is the reduction over the three-sphere S^3, which gives rise to the Salam-Sezgin SO(3) gauged supergravity [15]. By choosing other structure constants, corresponding to other three-dimensional Lie algebras, one can choose other group manifolds, some of which give rise to non-compact gaugings. In this section we describe the $D = 11$ supergravity theory and the reduction Ansatz that leads to the $D = 8$ gauged supergravity theories. We discuss the classification of the different $D = 3$ Lie algebras in appendix D and return to the issue of different $D = 8$ supergravities in section 3.

The fields of $N = 1, D = 11$ supergravity [20] are the Elfbein, a three-form potential and a 32-component Majorana gravitino:[1]

$$\text{11D}: \quad \left\{ \hat{\mathrm{e}}_{\hat{\mu}}{}^{\hat{\mathrm{a}}}, \hat{\mathrm{C}}_{\hat{\mu}\hat{\nu}\hat{\rho}}, \hat{\psi}_{\hat{\mu}} \right\}. \tag{2.1}$$

The bosonic part of the action and the supersymmetry up to bilinear fermions are given in appendix B.

To perform the dimensional reduction it is convenient to make an $8 + 3$ split of the 11-dimensional space-time: $x^{\hat{\mu}} = (x^\mu, z^m)$ with $\mu = (0, 1, \ldots 7)$ and $m = (1, 2, 3)$. We use the convention that space-time indices are $\hat{\mu} = (\mu, m)$ while the tangent indices are $\hat{a} = (a, i)$. Using a particular Lorentz frame the reduction Ansatz for the 11-dimensional fields is

$$\hat{e}_{\hat{\mu}}{}^{\hat{a}} = \begin{pmatrix} e^{-\frac{1}{6}\varphi} e_\mu{}^a & e^{\frac{1}{3}\varphi} L_m{}^i A^m{}_\mu \\ & \\ 0 & e^{\frac{1}{3}\varphi} L_n{}^i U^n{}_m \end{pmatrix} \tag{2.2}$$

and

$$\hat{C}_{abc} = e^{\frac{1}{2}\varphi} C_{abc}\,, \quad \hat{C}_{abi} = L_i{}^m B_{m\,ab}\,, \quad \hat{C}_{aij} = e^{-\frac{1}{2}\varphi}\,\epsilon_{mnp} L_i{}^m L_j{}^n\, V_a{}^p\,, \quad \hat{C}_{ijk} = e^{-\varphi}\epsilon_{ijk}\ell \tag{2.3}$$

[1]Our conventions are given in appendix A.

for the bosonic fields and

$$\hat{\psi}_{\hat{a}} = e^{\varphi/12}\left(\psi_a - \frac{1}{6}\Gamma_a\Gamma^i\lambda_i\right), \qquad \hat{\psi}_i = e^{\varphi/12}\lambda_i\,, \qquad \hat{\epsilon} = e^{-\varphi/12}\epsilon \tag{2.4}$$

for the fermions. Thus the full 8-dimensional field content consists of the following 128 + 128 field components (omitting spacetime indices on the potentials):

$$8\mathrm{D}: \quad \{e_\mu{}^a, L_m{}^i, \varphi, \ell, A^m, V^m, B_m, C, \psi_\mu, \lambda_i\}\,. \tag{2.5}$$

All these $D = 8$ fields are taken to be independent of z^m. We will now describe the quantities appearing in this reduction Ansatz.

The matrix $L_m{}^i$ describes the five-dimensional $\mathrm{SL}(3,\mathbb{R})/\,\mathrm{SO}(3)$ scalar coset of the internal space. It transforms under a global $\mathrm{SL}(3,\mathbb{R})$ acting from the left and a local SO(3) symmetry acting from the right. We take the following explicit representative, thus gauge fixing the local SO(3) symmetry:

$$L_m{}^i = \begin{pmatrix} e^{-\sigma/\sqrt{3}} & e^{-\phi/2+\sigma/2\sqrt{3}}\chi_1 & e^{\phi/2+\sigma/2\sqrt{3}}\chi_2 \\ 0 & e^{-\phi/2+\sigma/2\sqrt{3}} & e^{\phi/2+\sigma/2\sqrt{3}}\chi_3 \\ 0 & 0 & e^{\phi/2+\sigma/2\sqrt{3}} \end{pmatrix}, \tag{2.6}$$

which contains two dilatons, ϕ and σ, and three axions,[2] χ_1, χ_2 and χ_3. It is useful to define the local SO(3) invariant scalar matrix

$$\mathcal{M}_{mn} = -L_m{}^i L_n{}^j \eta_{ij}\,, \tag{2.7}$$

where $\eta_{ij} = -\mathbb{1}_3$ is the internal flat metric. Similarly, the two-dimensional $\mathrm{SL}(2,\mathbb{R})/\,\mathrm{SO}(2)$ scalar coset is parameterized by the dilaton φ and the axion ℓ via the local SO(2) invariant scalar matrix

$$\mathcal{W}_{IJ} = e^{\varphi}\begin{pmatrix} \ell^2 + e^{-2\varphi} & \ell \\ \ell & 1 \end{pmatrix}. \tag{2.8}$$

The only dependence on the internal coordinates z^m comes in via the $\mathrm{GL}(3,\mathbb{R})$ matrices $U^m{}_n$. These can be interpreted as the components of the 3 Maurer-Cartan 1-forms $\sigma^m \equiv U^m{}_n dz^n$ of some 3-dimensional Lie group. By definition they satisfy the Maurer-Cartan equations

$$d\sigma^m = -\frac{1}{2}f_{np}{}^m\sigma^n\wedge\sigma^p\,, \qquad f_{mn}{}^p = -2(U^{-1})^r{}_m(U^{-1})^s{}_n\,\partial_{[r}U^p{}_{s]}\,, \tag{2.9}$$

where the $f_{mn}{}^p$ are independent of z^m and form the structure constants of the group manifold.

[2]We call the scalars ℓ, χ_1, χ_2 and χ_3 axions and the scalars φ, ϕ and σ dilatons since (in the ungauged case) the axions only occur with a $D = 8$ spacetime derivative whereas the dilatons also occur without such a derivative.

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A subtlety which is not obvious from the analysis by Scherk and Schwarz is that only for traceless structure constants ($f_{mn}{}^{m} = 0$) one can reduce the action [21]. These cases lead to the class A gauged supergravities. For structure constants with non-vanishing trace ($f_{mn}{}^{m} \neq 0$) one has to resort to a reduction of the field equations. These cases lead to the class B gauged supergravities. Note that the embedding of the gauge group $\mathcal{G} \subset \mathrm{GL}(3,\mathbb{R})$ is described by

$$g_n{}^m = e^{i\lambda^k f_{kn}{}^m}\,, \tag{2.10}$$

where λ^k are the parameters of the gauge transformations. Therefore, in the case of a non-vanishing trace, the gauge group $\mathcal{G}$ is a subgroup of $\mathrm{GL}(3,\mathbb{R}) = \mathrm{SL}(3,\mathbb{R}) \otimes \mathrm{SO}(1,1)$ and not just $\mathrm{SL}(3,\mathbb{R})$.

Using a particular frame in the internal directions, the explicit coordinate dependence of the Maurer-Cartan one-forms corresponding to class A is given by

$$U^m{}_n = \begin{pmatrix} 1 & 0 & s_{1,3,2} \\ 0 & c_{2,3,1} & -c_{1,3,2}\, s_{2,3,1} \\ 0 & s_{3,2,1} & c_{1,3,2}\, c_{2,3,1} \end{pmatrix}, \qquad \det\, U \neq 1\,, \tag{2.11}$$

where we have used the following abbreviations ($a, b, c = 1, 2, 3$):

$$c_{a,b,c} \equiv \cos\left(\sqrt{\frac{1}{4} q_a q_b}\, z^c\right), \qquad s_{a,b,c} \equiv \sqrt{\frac{q_a}{q_b}} \sin\left(\sqrt{\frac{1}{4} q_a q_b}\, z^c\right). \tag{2.12}$$

This gives rise to structure constants $f_{mn}{}^p = \epsilon_{mnq} \mathsf{Q}^{pq}$ with $\mathsf{Q}^{pq} = \frac{1}{2}\,\mathrm{diag}(q_1, q_2, q_3)$. In section 3 we will explain that this actually suffices to study all class A gauged supergravities we obtain.

Note that the U-matrix is independent of z^3. It is always possible to choose a frame where z^3 is a manifest isometry. We distinguish the following three different cases:

1. The matrix Q is non-singular. In this case z^3 is the only manifest isometry. In the compact case we are dealing with the Salam-Sezgin case in which the group manifold is equal to S^3. The presence of the manifest z^3-isometry direction is related to the fact that S^3 can be viewed as a Hopf fibration over S^2. One consequence of this fact is that the $D = 8$ class A supergravities can also be obtained by reduction of the massless IIA theory. For instance, the Salam-Sezgin theory can alternatively be obtained by reduction of the massless IIA theory over S^2. The latter reduction naturally occurs in the context of the DW/QFT correspondence [6]. In the non-compact case the SO(3) gauging gets replaced by an SO(2,1) gauging. This case can be understood as an analytic continuation of the Salam-Sezgin theory or as a “non-compactification”[3] of $D = 11$ supergravity.

[3]As we will see in the next section in a non-compactification we have to discard an infinite factor in front of the action. Nevertheless, the procedure leads to a well-defined $D = 8$ gauged supergravity and, furthermore, can be used as a solution-generating transformation of $D = 11$ supergravity.

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2. The matrix Q is singular, e.g. $\mathsf{Q} = \frac{1}{2}\mathrm{diag}(0, q_2, q_3)$. In this case there is an additional isometry in the z^2-direction:

$$U^m{}_n = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sqrt{q_2/q_3}\sin\alpha \\ 0 & \sqrt{q_3/q_2}\sin\alpha & \cos\alpha \end{pmatrix}, \qquad \det\ U = 1\,, \tag{2.13}$$

with $\alpha = \sqrt{\frac{1}{4}q_2 q_3}z^1$. This means that the resulting $D = 8$ class A gauged supergravities can also be obtained by a reduction of the massless 9D theory.

3. The matrix Q is doubly-degenerate, e.g. $\mathsf{Q} = \frac{1}{2}\mathrm{diag}(0, 0, q_3)$. In this case the U-matrix is given by

$$U^m{}_n = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{1}{2}q_3 z^1 & 1 \end{pmatrix}, \qquad \det\ U = 1\,, \tag{2.14}$$

and again the resulting $D = 8$ class A gauged supergravity has its origin in the massless 9D theory.

The g.c.t.'s in the internal space generate symmetries in $D = 8$ dimensions. These g.c.t.'s are generated by the Killing vector

$$\hat{K}^m(\hat{x}) = -(U^{-1})^m{}_n(z)\lambda^n(x) + R^m{}_n z^n\,. \tag{2.15}$$

Upon reduction these correspond to

- $\mathrm{GL}(3,\mathbb{R})$ transformations with parameters $R^m{}_n$. These can be decomposed into $\mathrm{SL}(3,\mathbb{R})$ rotations, that act in the obvious way on all the fields that have m, n, p indices, and $\mathrm{SO}(1,1)$ rescalings.

- Gauge transformations with parameters λ^m or $\lambda_m{}^n \equiv -f_{mp}{}^n\lambda^p$ that act on all the fields that have m, n, p indices e.g.

$$\delta_\lambda L_m{}^i = -L_n{}^i\lambda_m{}^n = f_{mp}{}^n\lambda^p L_n{}^i\,, \tag{2.16}$$

  except for the Kaluza-Klein vectors $A^m{}_\mu$ that transform as gauge vectors,

$$\delta_\lambda A^m{}_\mu = \mathcal{D}_\mu\lambda^m \equiv \partial_\mu\lambda^m - f_{np}{}^m A^n{}_\mu\lambda^p\,, \tag{2.17}$$

  of the gauge group $\mathcal{G}$.

Performing the reduction of the 11-dimensional bosonic action (B.1) and supersymmetry variation of the gravitino (B.2) with the above reduction Ansatz, restricted to traceless structure constants, we find the class A $D = 8$ gauged supergravities described in the next section. Although in principle straightforward, we will not perform the explicit reduction for the class B theories in this paper.

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3. Gauged supergravities in $D = 8$

As discussed in the previous section, the different gauged supergravities can be obtained by a reduction of the 11-dimensional supergravity over different group manifolds. We restrict ourselves to gauge groups with traceless structure constants: $f_{mn}{}^n = 0$. For simplicity, we will only reduce the bosonic part of the action and consider the supersymmetry rules up to bilinears in the fermions. We give the full bosonic 8D action in appendix C. Here we consider the truncation that the $D = 11$ three-form potential is equal to zero. In this truncation the reduction of the 11D Einstein-Hilbert term gives rise to

$$S = \frac{1}{16\pi G_N^{(11)}} C_U \int d^8x \sqrt{|g|} \left[R + \frac{1}{4}\mathrm{Tr}\left(\mathcal{D}\mathcal{M}\mathcal{M}^{-1}\right)^2 + \frac{1}{2}(\partial\varphi)^2 - \frac{1}{4}e^{-\varphi} F^m \mathcal{M}_{mn} F^n - \mathcal{V} \right], \tag{3.1}$$

where the $\mathrm{SL}(3,\mathbb{R})/\,\mathrm{SO}(3)$ scalar matrix $\mathcal{M}$ is defined in (2.7), the potential $\mathcal{V}$ is given by

$$\mathcal{V} = \frac{1}{4}e^{-\varphi}\ \left[2\mathcal{M}^{nq} f_{mn}{}^p f_{pq}{}^m + \mathcal{M}^{mq}\mathcal{M}^{nr}\mathcal{M}_{ps} f_{mn}{}^p f_{qr}{}^s\right] \tag{3.2}$$

and C_U is defined by

$$C_U = \int dz^m \det(U^m{}_n)\,. \tag{3.3}$$

The integral that defines the factor C_U generally converges. It only diverges in the non-compact version of the Salam-Sezgin theory. The resulting $D = 8$ $\mathrm{SO}(2,1)$-gauged supergravity action is a "non-compactification" of $D = 11$ supergravity, see also the discussion in the previous section.

The covariant derivative $\mathcal{D}$ is always with respect to the gauge group $\mathcal{G}$ defined by the structure constants $f_{mn}{}^p$ and the gauge vectors A^m. Thus the gauge vector field strengths and the covariant derivative of the scalar coset read

$$F^m = 2\partial A^m - f_{np}{}^m A^n A^p\,, \qquad \mathcal{D}\mathcal{M}_{mn} = \partial\mathcal{M}_{mn} + 2f_{q(m}{}^p A^q \mathcal{M}_{n)p}\,. \tag{3.4}$$

The supersymmetry variations of the fermions read (in the truncation we are considering here)

$$\begin{aligned}
\delta\psi_\mu &= 2\partial_\mu\epsilon - \frac{1}{2}\not{\psi}_\mu\epsilon + \frac{1}{2}\not{Q}_\mu\epsilon - \frac{1}{2}e^{\varphi/2}\Gamma^m\left(\frac{1}{12}\Gamma_\mu\not{F}_m + F_{m\mu\nu}\Gamma^\nu\right)\epsilon + \frac{1}{24}e^{-\varphi/2} f_{ijk}\Gamma^{ijk}\Gamma_\mu\epsilon\,,\\
\delta\lambda_i &= -\not{P}_{ij}\Gamma^j\epsilon - \frac{1}{3}\not{\partial}\varphi\Gamma_i\epsilon + \frac{1}{4}e^{\varphi/2}\mathcal{M}_{in}\not{F}^n\epsilon - \frac{1}{4}e^{-\varphi/2}(2f_{ijk} - f_{jki})\Gamma^{jk}\epsilon\,,
\end{aligned} \tag{3.5}$$

where we have used the abbreviations $f_{ijk} \equiv L_i{}^m L_j{}^n L_{pk} f_{mn}{}^p$ and

$$P_{\mu ij} + Q_{\mu ij} \equiv L_i{}^m \mathcal{D}_\mu L_{mj}\,, \qquad \not{P}_{ij} \equiv P_{\mu ij}\Gamma^\mu\,, \qquad \not{Q}_\mu \equiv Q_{\mu ij}\Gamma^{ij}\,, \tag{3.6}$$

where P is symmetric and traceless and Q is antisymmetric.

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We observe that the massive deformations $f_{mn}{}^p$ come from the reduction over the group manifold. The choice $f_{mn}{}^p = 0$ is the ungauged case and corresponds to reduction over T^3 leading to the trivial gauge group $\mathrm{U}(1)^3$. The full supergravity theory has a global $\mathrm{SL}(3,\mathbb{R})\times\mathrm{SL}(2,\mathbb{R})$ symmetry group in the massless case. The $\mathrm{SL}(3,\mathbb{R})$ symmetry acts in the obvious way on the indices m, n, while the $\mathrm{SL}(2,\mathbb{R})$ symmetry is not a manifest symmetry of the action. Choosing non-vanishing structure constants modifies this symmetry group in the following way. The $\mathrm{SL}(2,\mathbb{R})$ symmetry is fully broken, while the $\mathrm{SL}(3,\mathbb{R})$ symmetry generically is broken due to the structure constants. Performing an $\mathrm{SL}(3,\mathbb{R})$ transformation has the effect of changing the structure constants via

$$f_{mn}{}^p \to f'_{mn}{}^p = R_m{}^q R_n{}^r (R^{-1})_s{}^p f_{qr}{}^s \,. \tag{3.7}$$

Only transformations that leave the structure constants invariant ($f_{mn}{}^p = f'_{mn}{}^p$) are unbroken by the massive deformations. This includes the infinitesimal gauge transformations (2.16).

The structure constants of all 3-dimensional Lie algebras can be parameterized by a symmetric matrix that we denote by Q^{mn} and which will play the role of mass matrix, and by a vector a_m satisfying $\mathsf{Q}^{mn} a_n = 0$ (see, e.g., [18]):

$$f_{mn}{}^p = \epsilon_{mnq}\mathsf{Q}^{qp} + 2\delta_{[m}{}^p a_{n]} \,. \tag{3.8}$$

The trace of the structure constants vanishes if and only if the vector vanishes, i.e. $a_m = 0$. Restricting to the class A gauged supergravities we can take $f_{mn}{}^p = \epsilon_{mnq}\mathsf{Q}^{qp}$ and all the different cases that we are going to consider will be characterized by a choice of mass matrix Q.

In terms of the mass matrix the potential reads

$$\mathcal{V} = -\frac{1}{2} e^{-\varphi} \left\{ [\mathrm{Tr}(\mathcal{M}\mathsf{Q})]^2 - 2\,\mathrm{Tr}(\mathcal{M}\mathsf{Q}\mathcal{M}\mathsf{Q}) \right\} . \tag{3.9}$$

The symmetric mass matrix Q has six different mass parameters. However, by applying symmetries of the massless 8D theory one can relate different choices of Q^{mn} by field redefinitions, via transformations as (3.7). We would like to use the $\mathrm{GL}(3,\mathbb{R})$ symmetry of the massless 8D theory.

Employing these symmetries we can transform $\mathsf{Q}^{mn} \to \pm(R^T\mathsf{Q}R)^{mn}$ with $R \in \mathrm{GL}(3,\mathbb{R})$. Now consider an arbitrary symmetric matrix Q^{mn} with eigenvalues λ_m and orthogonal eigenvectors $\vec{u}_m$. Taking $R = (c_1\vec{u}_1, c_2\vec{u}_2, c_3\vec{u}_3) \in \mathrm{GL}(3,\mathbb{R})$ with $c_i \neq 0$ we find that

$$\mathsf{Q}^{mn} \to \pm(R^T\mathsf{Q}R)^{mn} = \pm\,\mathrm{diag}(c_1{}^2\lambda_1, c_2{}^2\lambda_2, c_3{}^2\lambda_3) \,, \tag{3.10}$$

which is a minor extension of the Principal Axes theorem. Thus all cases with the same signature are related by field redefinitions. Without loss of generality we will use the freedom of field redefinitions to take

$$\mathsf{Q}^{mn} = \frac{1}{2}\,\mathrm{diag}(q_1, q_2, q_3) \,. \tag{3.11}$$

Bianchi	$\mathsf{Q} = \frac{1}{2}\mathrm{diag}$	Group
II	$(0, 0, q)$	Heisenberg
VI_0	$(0, -q, q)$	$\mathrm{ISO}(1, 1)$
VII_0	$(0, q, q)$	$\mathrm{ISO}(2)$
VIII	$(q, -q, q)$	$\mathrm{SO}(2, 1)$
IX	(q, q, q)	$\mathrm{SO}(3)$

Table 1: The different mass matrices and corresponding Bianchi classifications and gauge groups. The SO(3) result was previously obtained in [15].

The different 8D massive supergravities will arise from choosing all possible ranks and signatures for the mass matrix Q^{mn}.

Actually, this diagonalization plus the choice $a_m = (a, 0, 0)$ is the basis of the Bianchi classification of all real 3-dimensional Lie algebras from Bianchi type I to Bianchi type IX. Thus, each choice of mass matrix corresponds to a choice of Lie algebra and therefore of gauge group. Restricting to the class A theories we only consider the algebras with $a = 0$:[4]

$$\text{Bianchi types I, II, VI}_0,\ \text{VII}_0,\ \text{VIII, IX}\,. \tag{3.12}$$

All algebras with $a = 0$ are subalgebras of the Lie algebra of $\mathrm{SL}(3, \mathbb{R})$. For useful details about the Bianchi classification, see appendix D. The five nontrivial cases with $a = 0$ are given in table 1 while Bianchi type I corresponds to the massless case $\mathsf{Q} = 0$ and thus is an ungauged supergravity. This case corresponds to the abelian Lie algebra $\mathrm{U}(1)^3$.

4. The domain wall solutions

Having obtained the bosonic action and supersymmetry transformations of the $D = 8$ gauged maximal supergravities with gauge groups of class A, we now look for domain wall solutions that preserve half of the supersymmetry. For an earlier discussion of such solutions, see [22, 19]. We consider the following Ansatz:

$$ds^2 = g(y)^2 dx_7{}^2 - f(y)^2 dy^2\,, \qquad \mathcal{M} = \mathcal{M}(y)\,, \qquad \varphi = \varphi(y)\,, \qquad \epsilon = \epsilon(y)\,. \tag{4.1}$$

Our Ansatz only includes the metric and the scalars. All other fields are vanishing except the $\mathrm{SL}(2, \mathbb{R})/\,\mathrm{SO}(2)$ scalar ℓ which we have set constant. It turns out that there are no half-supersymmetric domain walls for non-constant ℓ.

[4]The sub-index 0 in Bianchi type VI_0 and Bianchi type VII_0 indicate that these class A Lie algebras can be obtained as the limit $a \to 0$ of the class B Bianchi type VI_a and Bianchi type VII_a Lie algebras (see appendix D).

We need to satisfy the Killing spinor equations

$$\begin{aligned}\delta\psi_\mu &= 2\partial_\mu\epsilon - \frac{1}{2}\not{\varphi}_\mu\epsilon + \frac{1}{2}\not{Q}_\mu\epsilon + \frac{1}{24}e^{-\varphi/2}f_{ijk}\Gamma^{ijk}\Gamma_\mu\epsilon = 0\,,\\ \delta\lambda_i &= -\not{P}_{ij}\Gamma^j\epsilon - \frac{1}{3}\not{\partial}\varphi\Gamma_i\epsilon - \frac{1}{4}e^{-\varphi/2}(2f_{ijk} - f_{jki})\Gamma^{jk}\epsilon = 0\,,\end{aligned} \tag{4.2}$$

where the Killing spinor satisfies the condition

$$(1+\Gamma_{y123})\epsilon = 0\,. \tag{4.3}$$

The indices $1,2,3$ refer to the internal group manifold directions.

The domain wall solutions we present below are valid both for a non-singular and a singular mass matrix Q. We find the following most general class A solution:

$$\begin{aligned}ds^2 &= H^{1/12}dx_7^2 - H^{-5/12}dy^2\,,\\ e^\varphi &= H^{1/4}\,, \qquad e^\sigma = H^{-1/(2\sqrt{3})}h_1^{\sqrt{3}/2}\,, \qquad e^\phi = H^{-1/2}h_1^{-1/2}(h_1h_2 - C_1^2)\,,\\ \chi_1 &= C_1h_1^{-1}\,, \qquad \chi_2 = \chi_1\chi_3 + C_2h_1^{-1}\,, \qquad \chi_3 = (C_1C_2 + C_3h_1)\left(h_1h_2 - C_1^2\right)^{-1}\,,\end{aligned} \tag{4.4}$$

where the dependence on the transverse coordinate y is governed by

$$\begin{aligned}H(y) &= h_1h_2h_3 - C_3^2h_1 - C_2^2h_2 - C_1^2h_3 - 2C_1C_2C_3\,,\\ h_1 &\equiv q_1y + C_4\,, \qquad h_2 \equiv q_2y + C_5\,, \qquad h_3 \equiv q_3y + C_6\,.\end{aligned} \tag{4.5}$$

The corresponding Killing spinor is quite intricate so we will not give it here. Note that the solution is given by three harmonic function h_1, h_2 and h_3. For this reason we call the general solution a triple domain wall.

The general solution has six integration constants $C_1,\ldots,C_6$. Note that the constants q_1, q_2 and q_3 should not be considered as parameters of the solution. They only serve to specify which of the five supergravities we are dealing with. The constants C_4, C_5 and C_6 are related to the positioning of the domain walls in the transverse space and q_1, q_2 and q_3 are their respective charges. The domain walls form a threshold bound state of n parallel domain walls, where n equals the rank of the mass matrix. It turns out that, provided that one of the charges q_1, q_2 or q_3 is non-zero, one can eliminate one of the constants C_4, C_5 or C_6 by a redefinition of the variable y. Therefore we effectively always end up with at most two constants.

The first three constants C_1, C_2 and C_3 can be understood to come from the following symmetry. The mass deformations do not break the full global $\mathrm{SL}(3,\mathbb{R})$; indeed, they gauge the 3-dimensional subgroup of $\mathrm{SL}(3,\mathbb{R})$ that leave the mass matrix Q invariant. Thus one can use the unbroken global subgroup to transform any solution,[5] introducing three constants. In our solution these correspond to C_1, C_2 and C_3 and thus these can be set to zero by fixing the $\mathrm{SL}(3,\mathbb{R})$ frame. From now on we will always assume the frame choice $C_1 = C_2 = C_3 = 0$ unless explicitly stated otherwise. This results in

$$\chi_1 = \chi_2 = \chi_3 = 0\,, \qquad \mathcal{M} = H^{-2/3}\mathrm{diag}(h_2h_3, h_1h_3, h_1h_2)\,, \qquad H = h_1h_2h_3\,. \tag{4.6}$$

[5]Note that one can not use the unbroken local subgroup of $\mathrm{SL}(3,\mathbb{R})$ (the gauge transformations) since this would induce non-vanishing gauge vectors and thus would be inconsistent with our Ansatz (4.1).

Bianchi	$\mathsf{Q} = \frac{1}{2}\text{diag}$	h_1	h_2	h_3	Δ	Uplift
II	$(0,0,q)$	C_4	C_5	$C+qy$	4	(5.8)
VII_0	$(0,q,q)$	C_4	$C+qy$	$C+qy$	$\times$	(5.6)
IX	(q,q,q)	$C+qy$	$C+qy$	$C+qy$	$-4/3$	(5.3)

Table 2: The single domain walls as truncations of the triple domain wall solution. We give the three possible truncations and the corresponding value of Δ [24]. Note that there exists no Δ-value in the Bianchi VII_0 case due to the vanishing of the potential. In the last column we indicate the equation where the uplifted solution to 11D is given.

This structure is very similar to that found in [23], upon which we will comment in the Conclusions. In this $\mathrm{SL}(3,\mathbb{R})$ frame the expression for the Killing spinor simplifies considerably and reads $\epsilon = H^{1/48}\epsilon_0$.

The triple domain wall can be truncated to double or single domain walls when restricting the constants C_4, C_5 and C_6. The single domain walls correspond to the situation where the positions of the parallel domain walls coincide. In table 2 we give the three possible truncations leading to single domain walls and the corresponding value of Δ as defined in [24]. The Bianchi II case was given in [22] and the Bianchi IX case in [6] (up to coordinate transformations). Note that the Bianchi VII_0 case can not be assigned a Δ-value since it has vanishing potential. The domain wall is carried by the non-vanishing massive contributions to the BPS equations. The same mechanism occurs in SO(2) gauged $D = 9$ supergravity [12].

The triple domain wall solution we found in this section can be interpreted as follows. One can view the $(0,0,q)$ solution, having one harmonic function, as the basic solution. The other solutions can then be obtained as threshold bound states of this solution with the $\mathrm{SL}(3,\mathbb{R})$-rotated solutions $(0,q,0)$ and $(q,0,0)$. This is clear at the level of the charges. We now see how, similarly, a composition rule at the level of the solutions can be established. One can thus view the solutions with a rank-1 mass matrix as building blocks for the general solution.

5. Uplifting to 11 dimensions

In this section we consider the uplifting of the triple domain wall solutions (4.4) to eleven dimensions. We find that upon uplifting, using the frame of (4.6), the triple domain wall solutions becomes a purely gravitational solutions with a metric of the form $d\hat{s}^2 = dx_7{}^2 - ds_4{}^2$, where

$$ds_4{}^2 = H^{-1/2}dy^2 + H^{1/2}\left(\frac{\sigma_1^2}{h_1} + \frac{\sigma_2^2}{h_2} + \frac{\sigma_3^2}{h_3}\right). \tag{5.1}$$

Here σ_1, σ_2 and σ_3 are the Maurer-Cartan 1-forms defined in (2.9), $H = h_1h_2h_3$ and the three harmonics h_1, h_2 and h_3 are defined in (4.5). The uplifted solutions are all 1/2 BPS.

The solutions (5.1) are cohomogeneity one solutions of different Bianchi types. The SO(3) expression of this 4-dimensional metric was obtained previously in the context of gravitational instanton solutions as self-dual metrics of Bianchi IX type with all directions unequal [25]. We find that the metric of [25] is related to a triple domain wall solution of 8-dimensional SO(3)-gauged supergravity. More recently, the Heisenberg, ISO(1,1) and ISO(2) cases and their relations to domain wall solutions were considered in [26, 27], whose results are related to ours via coordinate transformations.

It is remarkable that in all cases the uplifted solutions have metrics that contain a 7D Minkowski metric as a factor [19]. This does not happen for the uplift of domain walls in 4D and 7D gauged maximal supergravities [23]. In the following discussion we will focus on the 4-dimensional part of the eleven-dimensional metric since it characterizes the uplifted domain walls.

In section 4 we argued that for q_1, q_2 or q_3 non-zero one of the three constants C_4, C_5 or C_6 in the harmonic functions can be eliminated by a redefinition of the variable y. Without loss of generality we can take $q_3 > 0$. In that case the constant C_6 can be eliminated by the change of variables $y = \frac{1}{4}r^4 - C_6/q_3$ in the metric (5.1). If we also rescale the three charges by $q_m = 4\tilde{q}_m$ we obtain

$$ds_4{}^2 = (k_1 k_2 k_3)^{-1/2}\left[dr^2 + r^2(k_2 k_3 \sigma_1^2 + k_1 k_3 \sigma_2^2 + k_1 k_2 \sigma_3^2)\right], \tag{5.2}$$

where $k_i = \tilde{q}_i + s_i r^{-4}$ for $i = 1,2$, $s_i = C_{i+3} - q_i C_6/q_3$ and $k_3 = \tilde{q}_3$. As anticipated, the metric (5.2) depends only on two constant parameters s_1 and s_2, which are restricted by the gauge group dependent condition $-s_i < \tilde{q}_i r^4$, with $i = 1,2$, in order to satisfy the requirement $k_i(r) > 0$.

In general the metrics (5.2) have curvatures that both go to zero as r^{-6} for large r and diverge at $r = 0$, $r = (-s_1/\tilde{q}_1)^{1/4}$ and $r = (-s_2/\tilde{q}_2)^{1/4}$, producing incomplete metrics [28, 25]. There are two exceptions to this behavior. The first one corresponds to the case in which $s_1 = s_2 = 0$. The constants can take these values because $\tilde{q}_1$ and $\tilde{q}_2$ are non-zero and therefore this solution can be reached only for the non-degenerate cases. It is easy to see that for the SO(3) gauging ($\tilde{q}_1 = \tilde{q}_2 = \tilde{q}_3 = 1$) the metric is locally flat space-time

$$ds_4{}^2 = dr^2 + r^2((\sigma^1)^2 + (\sigma^2)^2 + (\sigma^3)^2), \tag{5.3}$$

where r is the radius of the 3-dimensional spheres. Notice that this is precisely the uplifting of the Bianchi IX single domain wall. This should correspond to the orbifold limit of the K3 manifold and therefore it is still only 1/2 BPS upon uplifting.

The second exception corresponds to the SO(3) gauging with $s_1 = s_2 \equiv s < 0$, and is known as the Eguchi-Hanson (EH), or Eguchi-Hanson II, metric [28] ($\tilde{q}_1 = \tilde{q}_2 = \tilde{q}_3 = 1$),

$$ds_4{}^2 = \left(1 + \frac{s}{r^4}\right)^{-1} dr^2 + r^2\left(\sigma_1^2 + \sigma_2^2\right) + \left(1 + \frac{s}{r^4}\right)\sigma_3^2. \tag{5.4}$$

In fact, the EH metric is the only complete and non-singular hyper-Kähler 4-metric admitting a tri-holomorphic SO(3) action. Its generic orbits are RP^3 [25, 29, 30].

Another case that we want to emphasize, although it is singular, is obtained in the SO(3) gauging by choosing $s_1 \equiv s \neq 0$ and $s_2 = 0$. This metric is called the Eguchi-Hanson

I (EH-I) metric [28] ($\tilde{q}_1 = \tilde{q}_2 = \tilde{q}_3 = 1$)

$$ds_4{}^2 = \left(1 + \frac{s}{r^4}\right)^{-1} (dr^2 + r^2\sigma_1^2) + \left(1 + \frac{s}{r^4}\right) (\sigma_2^2 + \sigma_3^2) \,. \tag{5.5}$$

It is possible to give similar expressions for the SO$(2,1)$ gauging.

The uplifted metrics for the singular mass matrices can be obtained directly from (5.2). As an example of a contraction we take $\tilde{q}_1 = 0$ in (5.2) and consider the special cases of the EH metrics (5.4) and (5.5), i.e. we take $s_1 = s_2 \equiv s < 0$ (EH-II) or $s_1 \equiv s \neq 0, s_2 = 0$ (EH-I). We thus obtain the contracted EH metrics with ISO(2) isometry in which the SO(3) orbits are flattened to ISO(2) orbits. We find that the expression for the contracted EH-I metric is given by ($\tilde{q}_2 = \tilde{q}_3 = 1$)

$$ds_4{}^2 = \left(\frac{s}{r^4}\right)^{-1/2} (dr^2 + r^2\sigma_1^2) + \left(\frac{s}{r^4}\right)^{1/2} (\sigma_2^2 + \sigma_3^2) \,, \tag{5.6}$$

while the expression for the contracted EH-II metric reads ($\tilde{q}_2 = \tilde{q}_3 = 1$)

$$\begin{aligned} ds_4{}^2 = & \left(\frac{s}{r^4}\left(1 + \frac{s}{r^4}\right)\right)^{-1/2} dr^2 + \left(\frac{s}{r^4}\left(1 + \frac{s}{r^4}\right)\right)^{1/2} \sigma_3^2 \; + \\ & + r^2 \left(\left(1 + \frac{r^4}{s}\right)^{1/2} \sigma_1^2 + \left(1 + \frac{r^4}{s}\right)^{-1/2} \sigma_2^2\right) . \end{aligned} \tag{5.7}$$

Notice that the contracted EH-I metric with ISO(2) isometry is precisely the 4-dimensional part of the uplifted metric for the Bianchi VII$_0$ single domain wall.

The metrics with Heisenberg isometry are obtained by a further contraction $\tilde{q}_1 = \tilde{q}_2 = 0$ in the metric (5.2). Again, among these metrics there is one special case that can also be obtained by a contraction of the contracted EH metric with isometry ISO(2). Notice that it is not possible to have a contracted EH-I metric with Heisenberg isometry since we must satisfy the condition $-s_i < \tilde{q}_i r^4$. The expression for the contracted EH metric with Heisenberg isometry is ($\tilde{q}_3 = 1$)

$$\begin{aligned} ds_4{}^2 &= \left(\frac{s}{r^4}\right)^{-1} dr^2 + r^2(\sigma_1^2 + \sigma_2^2) + \left(\frac{s}{r^4}\right) \sigma_3^2 \,, \\ &= \left(\frac{s}{r^4}\right)^{-1} dr^2 + r^2(dz_1{}^2 + dz_2{}^2) + \left(\frac{s}{r^4}\right) (dz_3 + 2z^1 dz_2)^2 \,, \end{aligned} \tag{5.8}$$

where $s_2 \equiv s$. This is the 4-dimensional part of the uplifted metric for the Bianchi II single domain wall This contraction was considered in [30].

6. Conclusions

In this paper we have considered two classes, A and B, of $D = 8$ gauged maximal supergravity theories. Class A contains supergravities with an action while the supergravities of class B only have equations of motion. Class A contains 5 supergravities corresponding to the following five different subgroups of SL$(3,\mathbb{R})$: SO(3), SO$(2,1)$, ISO(2), ISO$(1,1)$ and the Heisenberg subgroup. We have constructed a general half-supersymmetric triple domain-wall solution to these theories. It can be viewed as a threshold bound state of three parallel single domain walls. The uplifting of this solution to $D = 11$ dimensions leads to a purely gravitational solution whose metric is the direct product of a 7-dimensional Minkowski metric and a non-trivial 4-dimensional euclidean Ricci-flat metric [19]. The 4-metrics are solutions of 4-dimensional euclidean gravity. Among them we find generalizations of the Eguchi-Hanson solution to different (class A) Bianchi types.

The results of this paper are similar to the $D = 9$ case [12]: in both cases a $\mathrm{GL}(11-D, \mathbb{R})$ group ($D = 8, 9$) and its subgroups are the main characters. The group $\mathrm{GL}(11-D, \mathbb{R})$ appears naturally in (ungauged) maximal supergravities in D dimensions as part of its duality group since they can be obtained by toroidal compactification of 11-dimensional supergravity. It is natural to expect the existence of gauged supergravities associated to the subgroups of $\mathrm{GL}(11-D, \mathbb{R})$. Some cases are already well known, for instance the $D = 5$ maximal supergravities with gauge groups $\mathrm{SO}(6-l, l)$ (all of them subgroups of $\mathrm{SL}(6, \mathbb{R})$) constructed in [31, 32].[6] Gauged maximal supergravities in diverse dimensions have recently been investigated from a somewhat different point of view in [34].

An interesting outcome of our analysis is the existence, in $D = 8$ dimensions, of the generic triple domain wall solution (4.4). It can be interpreted as m parallel single domain walls where m is the rank of the mass matrix. For the gauging of $\mathrm{SO}(3)$, this result is similar to that of [23]. There the scalar content was the coset $\mathrm{SL}(n, \mathbb{R})/\mathrm{SO}(n)$ while we have the product of $n = 3$ with an additional $n = 2$ coset. Note that, in the gauged cases, our coset can not be reduced to the $\mathrm{SL}(3, \mathbb{R})/\mathrm{SO}(3)$ by truncation of the $\mathrm{SL}(2, \mathbb{R})/\mathrm{SO}(2)$ scalars. It is interesting that the structure of [23] extends to more general scalar contents and to the other Bianchi classes. It leads one to expect a similar n-tuple domain wall result in other dimensions. In fact, we verified that in $D = 9$ with scalar content $\mathrm{SO}(1,1) \times \mathrm{SL}(2, \mathbb{R})/\mathrm{SO}(2)$ the earlier results on domain wall solutions in gauged $D = 9$ supergravity [12] can be written as a generic double domain wall solution via coordinate transformations.

The relation between $D = 8$ domain wall solutions and gauged supergravities that we have discussed fits naturally in the domain wall/QFT correspondence scheme [5, 6]. As discussed in [6], taking the near-horizon limit of the D6-brane leads to the $D = 8$ $\mathrm{SO}(3)$ gauged supergravity. Taking the near-horizon limit of the direct reduction of the D6-brane to $D = 9$ dimensions leads to the $D = 8$ $\mathrm{ISO}(2)$-gauged supergravity. A further direct reduction to a 6-brane in $D = 8$ dimensions leads to the $D = 8$ Heisenberg gauged supergravity.

In this paper we mainly concentrated on the construction of the $D = 8$ class A gauged supergravities. We plan to investigate the class B theories as well. One difference with the class A theories is that the Maurer-Cartan 1-forms for traceful structure constants probably have no additional isometry. Therefore, in contrast to the class A case, these reductions cannot be reproduced by any known reduction of the massless IIA theory. Cohomogeneity one solutions of class B Bianchi type have been considered in the literature [26]. It would be interesting to see whether these solutions can be reduced to 1/2 BPS domain wall solutions of the corresponding class B $D = 8$ gauged supergravity.

[6] Other theories in different dimensions can also be constructed with an explicit symmetric mass matrix Q present [33].

It is interesting to note that the uplifting of the triple domain wall solution (4.4) does not lead to the most general 4-metric with SO(3) isometry. The complete non-singular SO(3)-invariant hyper-Kähler metrics in four dimensions are the Eguchi-Hanson, Taub-NUT and Atiyah-Hitchin metrics (for a useful discussion of these metrics see [35]). The absence of the Taub-NUT and Atiyah-Hitchin metrics in our analysis is related to the fact that only the (generalized) Eguchi-Hanson metric allows a covariantly constant spinor that is independent of the SO(3) isometry directions [36]. In performing the SS2 reduction we have assumed that our spinors are independent of the group manifold coordinates and this assumption is thus not compatible with the Taub-NUT and the Atiyah-Hitchin metrics. It would be interesting to see whether we can relax the SS2 procedure such that the Taub-NUT and Atiyah-Hitchin metrics also obtain a half-supersymmetric domain wall interpretation in $D = 8$ dimensions or whether we should view them as $D = 8$ domain walls with fully broken supersymmetry.

In the same spirit one can hope to extend the SS1 reduction, for example as applied in [12]. In that paper the spinors generally were transforming under the $\mathrm{SL}(2,\mathbb{R})$ duality symmetry and, consequently, the spinors were given dependence on the internal direction. However, for contracted group manifolds, our Ansätze with dependence on z_1 only, see (2.13), can be interpreted as a reduction from the massless 9D theory. In this case we have taken the spinors to be independent of the internal direction. We therefore have two reduction Ansätze that only differ in the fermionic sector. Therefore, both the SS1 and SS2 reduction procedure might be amenable to extension and it would be desirable to understand the differences between the resulting gauged supergravities.

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A. Conventions

Greek indices $\mu, \nu, \rho, \ldots$ denote world tensor indices while Latin $a, b, c \ldots$ indices are tangent space indices. We use hats for 11-dimensional objects and no hats for 8-dimensional objects. We symmetrize and antisymmetrize with weight one. Sometimes we use the following convention: when indices are not shown explicitly, we assume that all of them are world indices and all of them are completely antisymmetrized in the obvious order. This is similar to differential forms notation but the numerical factors differ.

We use mostly minus signature $(+-\cdots-)$.[7] η_{ab} is the Minkowski spacetime metric and the spacetime metric is $g_{\mu\nu}$. Lorentz and world indices are related by the Vielbeins $e_a{}^\mu$ and inverse Vielbeins $e_\mu{}^a$, that satisfy

$$e_a{}^\mu e_b{}^\nu g_{\mu\nu} = \eta_{ab}\,, \qquad e_\mu{}^a e_\nu{}^b \eta_{ab} = g_{\mu\nu}\,. \tag{A.1}$$

The spin connection ω_{abc} is defined by

$$\omega_{abc} = -\Omega_{abc} + \Omega_{bca} - \Omega_{cab}\,, \qquad \Omega_{ab}{}^c = e_a{}^\mu e_b{}^\nu \partial_{[\mu} e^c{}_{\nu]}\,. \tag{A.2}$$

The Riemann curvature tensor is given in terms of the spin connection by

$$R_{\mu\nu a}{}^b = 2\partial_{[\mu}\,\omega_{\nu]a}{}^b - 2\omega_{[\mu|a}{}^c\,\omega_{|\nu]c}{}^b\,. \tag{A.3}$$

The 11-dimensional gamma matrices satisfy the anticommutation relations

$$\{\hat\Gamma^{\hat a}, \hat\Gamma^{\hat b}\} = +2\hat\eta^{\hat a\hat b}\,. \tag{A.4}$$

We choose them to satisfy

$$\hat\Gamma^{\hat a\,\star} = -\hat\Gamma^{\hat a}\,, \qquad \hat\Gamma^0\,\hat\Gamma^{\hat a}\,\hat\Gamma^0 = \hat\Gamma^{\hat a\,\dagger}\,, \tag{A.5}$$

and thus are completely imaginary. With the definition $\bar\epsilon = i\epsilon^\dagger\hat\Gamma^0$ we can derive the properties

$$\bar{\hat\epsilon}\,\hat\Gamma^{\hat a_1\cdots\hat a_n}\,\hat\psi = (-1)^{n+[n/2]}\,\bar{\hat\psi}\,\hat\Gamma^{\hat a_1\cdots\hat a_n}\,\hat\epsilon\,, \tag{A.6}$$

and so the above bilinear is symmetric for $n = 0,3,4,7,8$ and antisymmetric for $n = 1,2,5,6,9,10$.

B. $D = 11$ supergravity

The bosonic part of the full $D = 11$ supergravity reads in our conventions

$$\hat S = \frac{1}{16\pi G_N^{(11)}}\int d^{11}\hat x\ \sqrt{|\hat g|}\ \left[\hat R(\hat\omega) - \frac{1}{2\cdot 4!}\hat G^{\,2} - \frac{1}{(144)^2}\frac{1}{\sqrt{|\hat g|}}\hat\epsilon\hat G\hat G\hat C\right], \tag{B.1}$$

where $\hat G = 4\,\partial\hat C$ and $G_N^{(11)}$ is the eleven-dimensional Newton constant. This action is invariant under the local supersymmetry transformations with parameter $\hat\epsilon$

$$\begin{aligned}
\delta_{\hat\epsilon}\hat e_{\hat\mu}{}^{\hat a} &= -\frac{i}{2}\bar{\hat\epsilon}\,\hat\Gamma^{\hat a}\,\hat\psi_{\hat\mu}\,,\\
\delta_{\hat\epsilon}\hat\psi_{\hat\mu} &= 2\hat\partial_{\hat\mu}\hat\epsilon - \frac{1}{2}\hat\omega_{\hat\mu}{}^{\hat a\hat b}\hat\Gamma_{\hat a\hat b}\hat\epsilon + \frac{i}{144}\left(\hat\Gamma^{\hat\alpha\hat\beta\hat\gamma\hat\delta}{}_{\hat\mu} - 8\hat\Gamma^{\hat\beta\hat\gamma\hat\delta}\hat\eta_{\hat\mu}{}^{\hat\alpha}\right)\hat\epsilon\,\hat G_{\hat\alpha\hat\beta\hat\gamma\hat\delta}\,,\\
\delta_{\hat\epsilon}\hat C_{\hat\mu\hat\nu\hat\rho} &= \frac{3}{2}\bar{\hat\epsilon}\,\hat\Gamma_{[\hat\mu\hat\nu}\,\hat\psi_{\hat\rho]}\,,
\end{aligned} \tag{B.2}$$

up to bilinears in fermions.

[7] All formulae in this paragraph are valid both without and with hats.

C. The 8-dimensional bosonic action

Restricting to gauge groups with traceless structure constants, $f_{mn}{}^n = 0$, the full bosonic 8-dimensional action reads

$$
\begin{aligned}
S = & \frac{1}{16\pi G_N^{(11)}} C_U \int d^8x \sqrt{|g_E|} \times \\
& \times \Bigg\{ R_E + \frac{1}{4}\mathrm{Tr}\left(\mathcal{D}\mathcal{M}\mathcal{M}^{-1}\right)^2 + \frac{1}{4}\mathrm{Tr}\left(\partial\mathcal{W}\mathcal{W}^{-1}\right)^2 - -\frac{1}{4}F^{I\,m}\mathcal{M}_{mn}\mathcal{W}_{IJ}F^{J\,n} + \\
& \quad + \frac{1}{2\cdot 3!}H_m\mathcal{M}^{mn}H_n - \frac{1}{2\cdot 4!}e^{\varphi}G^2 - \mathcal{V} - -\frac{1}{6^3\cdot 2^4}\frac{1}{\sqrt{|g_E|}}\epsilon \times \\
& \quad \times \Bigg[GG\ell - 8GH_mA^{2\,m} + 12G(\tilde{F}^m + \ell F^m)B_m - -8\epsilon^{mnp}H_mH_nB_p - 8G\partial\ell C - \\
& \quad - 16H_m(\tilde{F}^m + \ell F^m)C \Bigg] \Bigg\} ,
\end{aligned}
\tag{C.1}
$$

where we have made the following field strength definitions:

$$
\begin{aligned}
G &= 4\partial C + 6F^mB_m \,, & F^m &= 2\partial A^m - f_{np}{}^m A^nA^p \,, \\
H_m &= 3\mathcal{D}B_m + 3\epsilon_{mnp}F^nV^p \,, & \tilde{F}^m &= 2\mathcal{D}V^m - \epsilon^{mnp}f_{np}{}^qB_q \,,
\end{aligned}
\tag{C.2}
$$

The two vector field strengths form a doublet $F^{I\,m} = (F^m, \tilde{F}^m)$ with $I = 1, 2$. The $\mathrm{SL}(2,\mathbb{R})/\,\mathrm{SO}(2)$ and $\mathrm{SL}(3,\mathbb{R})/\,\mathrm{SO}(3)$ scalar cosets are defined in (2.8) and (2.7), respectively. The potential $\mathcal{V}$ reads

$$
\mathcal{V} = \frac{1}{4}e^{-\varphi}\left[2\mathcal{M}^{nq}f_{mn}{}^pf_{pq}{}^m + \mathcal{M}^{mq}\mathcal{M}^{nr}\mathcal{M}_{ps}f_{mn}{}^pf_{qr}{}^s\right] .
\tag{C.3}
$$

The supersymmetric transformation rules in eight dimensions are:

$$
\begin{aligned}
\delta e_\mu{}^a &= -\frac{i}{2}\bar{\epsilon}\Gamma^a\psi_\mu \,, \\
\delta\psi_\mu &= 2\partial_\mu\epsilon - \frac{1}{2}\not{\psi}_\mu\epsilon + \frac{1}{2}L_{[i|}{}^m\mathcal{D}_\mu L_{m|j]}\Gamma^{ij}\epsilon + \frac{1}{24}e^{-\varphi/2}f_{ijk}\Gamma^{ijk}\Gamma_\mu\epsilon + \\
&\quad + \frac{1}{24}e^{\varphi/2}\Gamma^iL_i{}^m(\Gamma_\mu{}^{\nu\rho} - 10\delta_\mu{}^\nu\Gamma^\rho)F_{m\nu\rho}\epsilon + \frac{1}{2}e^{-\varphi}\partial_\mu\ell\epsilon + \\
&\quad + \frac{i}{96}e^{\varphi/2}(\Gamma_\mu{}^{\nu\rho\delta\epsilon} - 4\delta_\mu{}^\nu\Gamma^{\rho\delta\epsilon})G_{\nu\rho\delta\epsilon}\epsilon + \frac{i}{36}\Gamma^iL_i{}^m(\Gamma_\mu{}^{\nu\rho\delta} - 6\delta_\mu{}^\nu\Gamma^{\rho\delta})H_{m\nu\rho\delta}\epsilon + \\
&\quad + \frac{i}{48}e^{-\varphi/2}\Gamma^i\Gamma^jL_i{}^mL_j{}^n(\Gamma_\mu{}^{\nu\rho} - 10\delta_\mu{}^\nu\Gamma^\rho)(F_{mn\nu\rho} + \ell\varepsilon_{mnp}F^p{}_{\nu\rho})\epsilon \,, \\
\delta\lambda_i &= \frac{1}{2}L_i{}^mL^{jn}\not{\mathcal{D}}\mathcal{M}_{mn}\Gamma_j\epsilon - \frac{1}{3}\not{\partial}\varphi\Gamma_i\epsilon - \frac{1}{4}e^{-\varphi/2}(2f_{ijk} - f_{jki})\Gamma^{jk}\epsilon + \\
&\quad + \frac{1}{4}e^{\varphi/2}L_i{}^m\mathcal{M}_{mn}\not{F}^n\epsilon + \frac{i}{144}e^{\varphi/2}\Gamma_i\not{G}\epsilon + \frac{i}{36}(2\delta_i{}^j - \Gamma_i{}^j)L_j{}^m\not{H}_m\epsilon \\
&\quad + \frac{i}{24}e^{-\varphi/2}\Gamma^jL_j{}^mL_k{}^n(3\delta_i{}^k - \Gamma_i{}^k)(\not{F}_{mn} + \ell\varepsilon_{mnp}\not{F}^p)\epsilon + \frac{1}{3}e^{-\varphi}\Gamma_i\not{\partial}\ell\epsilon \,,
\end{aligned}
$$

$$
\begin{aligned}
\delta A_\mu{}^m &= -\frac{i}{2}e^{-\varphi/2}L_i{}^m\bar\epsilon\left(\Gamma^i\psi_\mu - \Gamma_\mu\left(\eta^{ij}-\frac{1}{6}\Gamma^i\Gamma^j\right)\lambda_j\right),\\
\delta V_\mu{}^m &= -\frac{i}{2}e^{\varphi/2}L_i{}^m\bar\epsilon\left(\Gamma^i\psi_\mu + \Gamma_\mu\left(\eta^{ij}-\frac{5}{6}\Gamma^i\Gamma^j\right)\lambda_j\right) - \ell\delta A_\mu{}^m,\\
\delta B_{\mu\nu m} &= L_m{}^i\bar\epsilon\left(\Gamma_{i[\mu}\psi_{\nu]} + \frac{1}{6}\Gamma_{\mu\nu}(3\delta_i{}^j - \Gamma_i\Gamma^j)\lambda_j\right) - 2\varepsilon_{mnp}\delta A_\mu{}^n V_\nu{}^p,\\
\delta C_{\mu\nu\rho} &= \frac{3}{2}e^{-\varphi/2}\bar\epsilon\Gamma_{[\mu\nu}\left(\psi_{\rho]} - \frac{1}{6}\Gamma_{\rho]}\Gamma^i\lambda_i\right) - 3\delta A_{[\mu}{}^m B_{\nu\rho]m},\\
L_i{}^n\delta L_{nj} &= \frac{i}{4}e^{\varphi/2}\bar\epsilon\left(\Gamma_i\delta_j{}^k + \Gamma_j\delta_i{}^k - \frac{2}{3}\eta_{ij}\Gamma^k\right)\lambda_k,\\
\delta\varphi &= -\frac{i}{2}\bar\epsilon\Gamma^i\lambda_i,\\
\delta\ell &= -\frac{i}{2}e^{\varphi}\bar\epsilon\Gamma^i\lambda_i,
\end{aligned}
\tag{C.4}
$$

D. The Bianchi classification

In this appendix we will discuss the Bianchi classification of three-dimensional Lie algebras. We will also show how different algebras are related via analytic continuation or group contraction. We compare our results with the CSO(p,q,r) notation which is often used in the supergravity literature.

We assume that the generators of the three-dimensional Lie group satisfy the commutation relations ($m = 1,2,3$)

$$[T_m, T_n] = f_{mn}{}^p T_p\,, \tag{D.1}$$

with constant structure coefficients $f_{mn}{}^p$ subject to the Jacobi identity $f_{m[n}{}^p f_{qr]p} = 0$. For three-dimensional Lie groups the structure constants have nine components and can be conveniently parameterized by

$$f_{mn}{}^p = \epsilon_{mnq}\mathsf{Q}^{pq} + 2\delta_{[m}{}^p a_{n]}\,. \tag{D.2}$$

Here Q^{pq} is a symmetric matrix. The Jacobi identity implies $\mathsf{Q}^{pq}a_q = 0$. Having $a_q = 0$ corresponds to an algebra with traceless structure constants: $f_{mn}{}^n = 0$.

Of course Lie algebras are only defined up to changes of basis $T_m \to R_m{}^n T_n$. This can always be used [18, 37] to transform Q^{pq} into a diagonal form and a_q to have only one component. We will take $\mathsf{Q}^{pq} = \frac{1}{2}\,\mathrm{diag}(q_1,q_2,q_3)$ and $a_q = (a,0,0)$. The commutation relations then take the form

$$[T_1,T_2] = \frac{1}{2}q_3T_3 - aT_2\,,\qquad [T_2,T_3] = \frac{1}{2}q_1T_1\,,\qquad [T_3,T_1] = \frac{1}{2}q_2T_2 + aT_3\,. \tag{D.3}$$

The different three-dimensional Lie algebras have been classified and are given in table 3. There are 11 different algebras, two of which are a one-parameter family. Of these only SO(3) and SO(2,1) are simple while the rest are all non-semi-simple [37, 38]. Note that for $a \neq 0$ the rank of Q can not exceed two due to the Jacobi identity.

Class	Bianchi	a	(q_1, q_2, q_3)	Group	$\mathrm{CSO}(p,q,r)$
A	I	0	$(0,0,0)$	$\mathrm{U}(1)^3$	$(0,0,3)$
A	II	0	$(0,0,1)$	Heisenberg	$(1,0,2)$
A	VI_0	0	$(0,-1,1)$	$\mathrm{ISO}(1,1)$	$(1,1,1)$
A	VII_0	0	$(0,1,1)$	$\mathrm{ISO}(2)$	$(2,0,1)$
A	VIII	0	$(1,-1,1)$	$\mathrm{SO}(2,1)$	$(2,1,0)$
A	IX	0	$(1,1,1)$	$\mathrm{SO}(3)$	$(3,0,0)$
B	V	1	$(0,0,0)$		
B	IV	1	$(0,0,1)$		
B	III	1	$(0,-1,1)$		
B	VI_a	a	$(0,-1,1)$		
B	VII_a	a	$(0,1,1)$		

Table 3: The different three-dimensional Lie algebras in terms of components of their structure constants and the Bianchi and $\mathrm{CSO}(p,q,r)$ classification. Note that there are two one-parameter families VI_a and VII_a with special case VI_0, VII_0 and VI_1=III.

We will now show relations between all algebras of Class A, i.e. having $a = 0$. Our starting point will be $\mathrm{SO}(3)$. Its generators take, in our basis with $\mathsf{Q} = \frac{1}{2}\,\mathrm{diag}(1,1,1)$, the form

$$T_1 = \frac{1}{2}\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad T_2 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}, \qquad T_3 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}. \tag{D.4}$$

One can obtain the other algebras with $a = 0$ from these $\mathrm{SO}(3)$ generators by the analytic continuation and/or contraction. Define the operations A_1 (analytic continuation) and C_1 (contraction) by

$$T_2 \to \lambda^{-1}\,T_2\,, \qquad T_3 \to \lambda^{-1}\,T_3\,, \tag{D.5}$$

with $\lambda = i$ for A_1 and $\lambda \to 0$ for C_1. Its effect on the parameters of the algebra reads

$$\mathsf{Q} = \frac{1}{2}\,\mathrm{diag}(q_1, q_2, q_3) \to \mathsf{Q} = \frac{1}{2}\,\mathrm{diag}(\lambda^2 q_1, q_2, q_3)\,. \tag{D.6}$$

Thus from $\mathrm{SO}(3)$ one can obtain $\mathrm{SO}(2,1)$ by an A operation and $\mathrm{ISO}(2)$ by a C operation. Similarly, the other Class A algebras are related by various analytic continuations and contractions, as shown in figure 4.

It is instructive to compare the discussion of the previous paragraph with the $\mathrm{CSO}(p,q,r)$ notation which is often used in the supergravity literature, see e.g. [39, 40].

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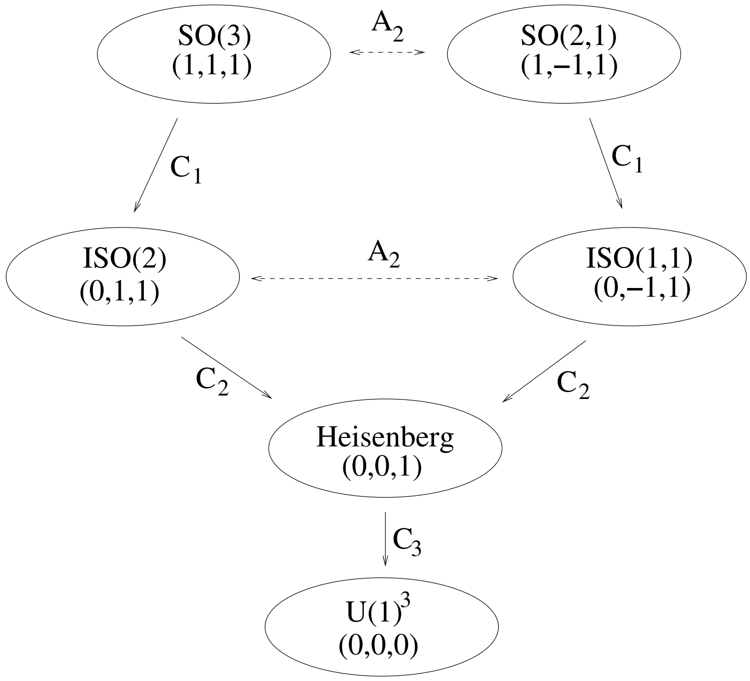

Table 4: Relations between groups associated to the 3D Class A Lie algebras. The boxes give the groups and the components $\mathsf{Q}^{mn} = \frac{1}{2}\mathrm{diag}(q_1, q_2, q_3)$ of the structure constants. The arrows give the operations: the dashed arrow corresponds to the reversible analytic continuation, the solid arrow to the irreversible group contraction. These analytic continuations and contractions are defined in (D.7) and (D.8).

In our case $p + q + r = 3$ but the $\mathrm{CSO}(p, q, r)$ classification of contracted algebras is valid more generally. The $\mathrm{CSO}(p, q, r)$ group is a group contraction of $\mathrm{SO}(p + r, q)$ and can be obtained as follows. One defines the starting point $\mathrm{CSO}(p, q, 0) = \mathrm{SO}(p, q)$. The effect of analytic continuation in one of the p directions is

$$A_p : \qquad \mathrm{CSO}(p, q, r) \to \mathrm{CSO}(p - 1, q + 1, r)\,, \tag{D.7}$$

while the effect of contraction is

$$C_p : \qquad \mathrm{CSO}(p, q, r) \to \mathrm{CSO}(p - 1, q, r + 1)\,. \tag{D.8}$$

This defines all Class A algebras in terms of the $\mathrm{CSO}(p, q, r)$ classification, as shown in table 3. These can all be obtained from the semi-simple algebras $\mathrm{SO}(3)$ or $\mathrm{SO}(2, 1)$ by various contractions. Using the fact that $\mathrm{CSO}(p, q, r) \sim \mathrm{CSO}(q, p, r)$ one can see that Class A exhausts the possibilities of distributing p, q, r subject to $p + q + r = 3$.

References

[1] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

JHEP06(2003)038

[2] Supernova Search Team collaboration, A.G. Riess et al., *Observational evidence from supernovae for an accelerating universe and a cosmological constant*, *Astron. J.* **116** (1998) 1009 [astro-ph/9805201].

[3] P. Fré, M. Trigiante and A. Van Proeyen, *Stable de Sitter vacua from $N = 2$ supergravity*, *Class. and Quant. Grav.* **19** (2002) 4167 [hep-th/0205119].

[4] M. de Roo, D.B. Westra and S. Panda, *de Sitter solutions in $N = 4$ matter coupled supergravity*, *J. High Energy Phys.* **02** (2003) 003 [hep-th/0212216].

[5] N. Itzhaki, J.M. Maldacena, J. Sonnenschein and S. Yankielowicz, *Supergravity and the large-N limit of theories with sixteen supercharges*, *Phys. Rev.* **D 58** (1998) 046004 [hep-th/9802042].

[6] H.J. Boonstra, K. Skenderis and P.K. Townsend, *The domain wall/QFT correspondence*, *J. High Energy Phys.* **01** (1999) 003 [hep-th/9807137].

[7] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [hep-ph/9905221].

[8] L. Randall and R. Sundrum, *An alternative to compactification*, *Phys. Rev. Lett.* **83** (1999) 4690 [hep-th/9906064].

[9] R. Kallosh, A.D. Linde, S. Prokushkin and M. Shmakova, *Gauged supergravities, de Sitter space and cosmology*, *Phys. Rev.* **D 65** (2002) 105016 [hep-th/0110089].

[10] P.K. Townsend, *Quintessence from M-theory*, *J. High Energy Phys.* **11** (2001) 042 [hep-th/0110072].

[11] P.M. Cowdall, *Novel domain wall and Minkowski vacua of $D = 9$ SO(2) gauged supergravity*, *Nucl. Phys.* **B 600** (2001) 81.

[12] E. Bergshoeff, U. Gran and D. Roest, *Type IIB seven-brane solutions from nine-dimensional domain walls*, *Class. and Quant. Grav.* **19** (2002) 4207 [hep-th/0203202].

[13] H. Nishino and S. Rajpoot, *Gauged $N = 2$ supergravity in nine-dimensions and domain wall solutions*, *Phys. Lett.* **B 546** (2002) 261 [hep-th/0207246].

[14] E. Bergshoeff, T. de Wit, U. Gran, R. Linares and D. Roest, *(Non-)abelian gauged supergravities in nine dimensions*, *J. High Energy Phys.* **10** (2002) 061 [hep-th/0209205].

[15] A. Salam and E. Sezgin, *$D = 8$ supergravity*, *Nucl. Phys.* **B 258** (1985) 284.

[16] J. Scherk and J.H. Schwarz, *How to get masses from extra dimensions*, *Nucl. Phys.* **B 153** (1979) 61.

[17] J. Scherk and J.H. Schwarz, *Spontaneous breaking of supersymmetry through dimensional reduction*, *Phys. Lett.* **B 82** (1979) 60.

[18] R.M. Wald, *General relativity*, Chicago University Press, Chicago 1984.

[19] R. Hernandez and K. Sfetsos, *An eight-dimensional approach to G_2 manifolds*, *Phys. Lett.* **B 536** (2002) 294 [hep-th/0202135].

[20] E. Cremmer, B. Julia and J. Scherk, *Supergravity theory in 11 dimensions*, *Phys. Lett.* **B 76** (1978) 409.

[21] J.M. Pons and L.C. Shepley, *Dimensional reduction and gauge group reduction in Bianchi-type cosmology*, *Phys. Rev.* **D 58** (1998) 024001 [gr-qc/9805030].

[22] P.M. Cowdall, H. Lu, C.N. Pope, K.S. Stelle and P.K. Townsend, *Domain walls in massive supergravities*, *Nucl. Phys.* **B 486** (1997) 49 [hep-th/9608173].

[23] I. Bakas, A. Brandhuber and K. Sfetsos, *Domain walls of gauged supergravity, M-branes and algebraic curves*, *Adv. Theor. Math. Phys.* **3** (1999) 1657 [hep-th/9912132].

[24] H. Lu, C.N. Pope, E. Sezgin and K.S. Stelle, *Stainless super p-branes*, *Nucl. Phys.* **B 456** (1995) 669 [hep-th/9508042].

[25] V.A. Belinsky, G.W. Gibbons, D.N. Page and C.N. Pope, *Asymptotically euclidean Bianchi IX metrics in quantum gravity*, *Phys. Lett.* **B 76** (1978) 433.

[26] G.W. Gibbons and P. Rychenkova, *Single-sided domain walls in M-theory*, *J. Geom. Phys.* **32** (2000) 311 [hep-th/9811045].

[27] I.V. Lavrinenko, H. Lu and C.N. Pope, *Fibre bundles and generalised dimensional reductions*, *Class. and Quant. Grav.* **15** (1998) 2239 [hep-th/9710243].

[28] T. Eguchi and A.J. Hanson, *Asymptotically flat selfdual solutions to euclidean gravity*, *Phys. Lett.* **B 74** (1978) 249.

[29] G.W. Gibbons and C.N. Pope, *The positive action conjecture and asymptotically euclidean metrics in quantum gravity*, *Commun. Math. Phys.* **66** (1979) 267.

[30] G.W. Gibbons, H. Lu, C.N. Pope and K.S. Stelle, *Supersymmetric domain walls from metrics of special holonomy*, *Nucl. Phys.* **B 623** (2002) 3 [hep-th/0108191].

[31] M. Günaydin, L.J. Romans and N.P. Warner, *Gauged $N = 8$ supergravity in five-dimensions*, *Phys. Lett.* **B 154** (1985) 268.

[32] M. Pernici, K. Pilch and P. van Nieuwenhuizen, *Gauged $N = 8$ $D = 5$ supergravity*, *Nucl. Phys.* **B 259** (1985) 460.

[33] N. Alonso-Alberca and T. Ortín, *Gauged/massive supergravities in diverse dimensions*, *Nucl. Phys.* **B 651** (2003) 263 [hep-th/0210011].

[34] B. de Wit, H. Samtleben and M. Trigiante, *On lagrangians and gaugings of maximal supergravities*, *Nucl. Phys.* **B 655** (2003) 93 [hep-th/0212239].

[35] I. Bakas and K. Sfetsos, *Toda fields of* SO(3) *hyper-Kähler metrics and free field realizations*, *Int. J. Mod. Phys.* **A 12** (1997) 2585 [hep-th/9604003].

[36] J.P. Gauntlett, G.W. Gibbons, G. Papadopoulos and P.K. Townsend, *Hyper-Kähler manifolds and multiply intersecting branes*, *Nucl. Phys.* **B 500** (1997) 133 [hep-th/9702202].

[37] J. Schirmer, *Hamiltonian reduction of Bianchi cosmologies*, *Class. and Quant. Grav.* **12** (1995) 1099 [gr-qc/9408008].

[38] M. Hamermesh, *Group theory and its application to physical problems*, Dover Publication, New York 1989.

[39] C.M. Hull, *The minimal couplings and scalar potentials of the gauged $N = 8$ supergravities*, *Class. and Quant. Grav.* **2** (1985) 343.

[40] C.M. Hull, *New gauged $N = 8$, $D = 4$ supergravities*, hep-th/0204156.